Near-Perfect Correlation of the Resistance Components of Mesoscopic Samples at the Quantum Hall Regime

E. Peled,[1] D. Shahar,[1] Y. Chen,[2] E. Diez,[2,*] D. L. Sivco,[3] and A. Y. Cho[3]

[1]Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot 76100, Israel

[2]Department of Electrical Engineering, Princeton University, Princeton, New Jersey 08544, USA

[3]Bell Laboratories, Lucent Technologies, 600 Mountain Avenue, Murray Hill, New Jersey 07974, USA

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We study the four-terminal resistance fluctuations of mesoscopic samples near the transition between the $\nu = 2$ and the $\nu = 1$ quantum Hall states. We observe near-perfect correlations between the fluctuations of the longitudinal and Hall components of the resistance. These correlated fluctuations appear in a magnetic-field range for which the two-terminal resistance of the samples is quantized. We discuss these findings in light of edge-state transport models of the quantum Hall effect. We also show that our results lead to an ambiguity in the determination of the width of quantum Hall transitions.

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When placed in strong magnetic fields (Bs), two-dimensional electron systems can display a series of states known as the quantum hall effect (QHE) [1]. These states display a remarkable universality: irrespective of many of the system's properties such as geometry and disorder strength, its Hall resistance (R_H) exhibits exact quantization at h/ie^2 (h is Planck's constant, e is the charge of the electron, and i is an integer), while its longitudinal resistance (R_L) vanishes.

When the size of the samples becomes smaller, approaching the mesoscopic regime, the features of the QHE begin to diminish. In addition, a pattern of reproducible fluctuations appears, whose magnitude increases as the sample size and the temperature (T) decrease. Near $B = 0$ these are the well-known universal conductance fluctuations [2] famous for the universality of their amplitude, which is close to e^2/h. In the quantum Hall (QH) regime the understanding of the fluctuations is not as complete, despite the large number of experimental [3–15] and theoretical [16–23] studies. In particular, the amplitude of the fluctuations in this regime shows a distinct B dependence and is not universal.

In this Letter we report on the observation of a new type of universal behavior of the fluctuations in the QH regime. We have measured R_L and R_H under the conditions of the QHE, in mesoscopic samples for which finite-size effects are dominant. Our samples display reproducible resistance fluctuations that are cooldown, as well as contact configuration, specific. We found that there are near-perfect correlations between the fluctuations measured in R_L and those measured in R_H. Specifically, in the vicinity of the transition between the $\nu = 2$ and the $\nu = 1$ QH states, we find that

$$R_L + R_H = h/e^2 \tag{1}$$

over a wide range of B. We trace the origin of these correlations to the quantization, over the same B range, of the two-terminal resistance of the sample (R_{2t}). The link between the sum $R_L + R_H$ and R_{2t} is in accordance with the transport model of Streda *et al.* [24], that combines the Landauer formulation for conductance with the existence of electronic edge states at high Bs [25,26]. Finally, we demonstrate that our findings reveal an ambiguity in the determination of the width of QH transitions, a property that is material to the subject of scaling and universality in QH transitions.

The samples we used (T2Cm2, T2Cm20, and T2Cm100) were made from a InGaAs/InAlAs wafer containing a 200 Å quantum well. The short-range scattering in the wafer leads to the formation of a low-mobility, low-density two-dimensional electron system, after illumination with an LED. Our samples have an average density $n_s = 1.15 \times 10^{11}$ cm^{-2} and average mobility $\mu = 16\,600$ cm^2/V sec, limiting our study to the integer QHE. We have defined three Hall-bar samples, wet-etched with the same aspect ratio [see Fig. 1(a)], but with lithographic widths of $W = 2$, 20, and 100 μm. To ensure maximum uniformity, the three samples were prepared on the same chip within 2 mm of each other. The black areas in Fig. 1(a) represent Au-Ge-Ni alloyed contacts that were designed to reach the edges of the Hall bars. The samples were cooled in a dilution refrigerator with a base T of 10 mK, at which all of the data presented here were taken. Four- and two-terminal measurements were done using standard ac lock-in techniques with a frequency of 3.17 Hz and an excitation current of 1 nA. The value of 1 nA for the current was chosen to avoid electron heating. At higher current values ($I \geq 10$ nA) we find evidence for heating: the resistance fluctuations diminish in magnitude and the width of the QH transitions increases.

We begin the description of our data by presenting, in Fig. 1(b), B traces of R_L and R_H for the 2 μm Hall bar in the vicinity of the transition between the $\nu = 2$ and the $\nu = 1$ QH states. Referring to the diagram in Fig. 1(a),

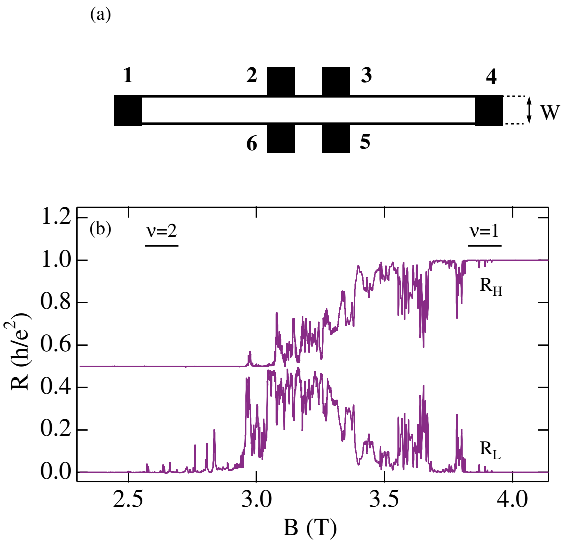

FIG. 1 (color). (a) Geometry of the Hall-bar samples. The black areas represent Au-Ge-Ni contacts. The separation of the current and voltage contacts are $12 \times W$ and $2 \times W$, respectively. (b) R_L and R_H vs B of the 2 μm Hall bar in the vicinity of the $\nu = 2$-1 transition, $T = 10$ mK.

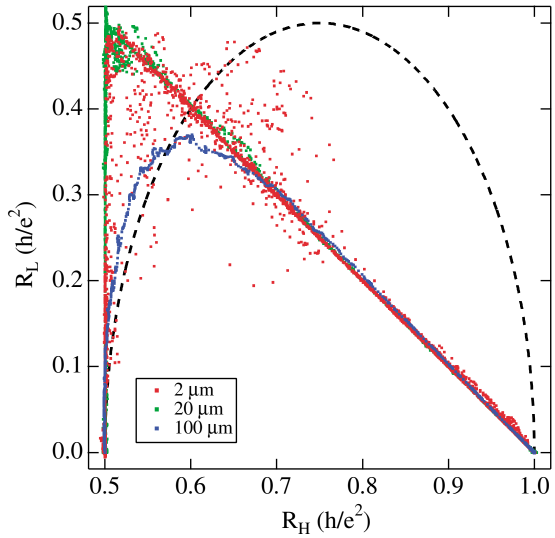

FIG. 2 (color). R_L vs R_H for the 2, 20, and 100 μm Hall bars in the vicinity of the $\nu = 2$-1 transition, $T = 10$ mK. Dashed line: the theoretical semicircle relation for a macroscopic sample with the same aspect ratio as our samples.

and using the standard notation $R_{ij,kl} = V_{kl}/I_{ij}$, where V_{kl} is the voltage difference between probes k and l and I_{ij} is the current between probes i and j, the data we show are $R_L = R_{14,65}$ and $R_H = R_{14,53}$. Despite the small size of the sample, the $\nu = 1$ ($B > 3.9$ T) and $\nu = 2$ ($B < 2.55$ T) QH states are clearly seen, evident by the quantization of R_H and the corresponding vanishing of R_L.

The finite size of the sample is manifested by the appearance, in the transition region, of large, reproducible, fluctuations in both R_L and R_H. As seen in previous studies of mesoscopic samples in the QH regime, these noiselike fluctuations maintain their pattern as long as the sample is kept cold, and diminish in magnitude as T is increased. A new fluctuation pattern is found each time the sample is temperature cycled.

The central finding of our work is the existence, on the $\nu = 1$ side of the transition [$B = 3.1$–3.9 T in Fig. 1(b)], of near-perfect correlations between the fluctuations of R_L and those of R_H. Graphically, we observe that for each peak in R_L there corresponds a dip in R_H of nearly equal magnitude, and vice versa. This holds for almost all the fine details of the fluctuation patterns. While such correlations could arise from mixing of the resistance components, it is unlikely that this is the case in our work since the correlations are limited to a specific range of B and do not show up at either low B or at the $\nu = 2$ side of the transition.

Mathematically, the correlations we observe can be conveniently expressed by the simple relation of Eq. (1). To see this dependence more clearly we plot, in Fig. 2, R_L vs R_H for all three samples. In this unconventional plot [26] each dot represents one (R_H, R_L) data pair from B traces such as those in Fig. 1(b). Focusing on the data from the 2 μm sample of Fig. 1(b) (red dots in Fig. 2) we see that, aside from some scatter, the dots fall into two ordered groups: a diagonal line stretching from ($0.5h/e^2$, $0.5h/e^2$) to (h/e^2, 0) and a vertical line at $R_H = 0.5h/e^2$. The diagonal line corresponds to $R_L + R_H = h/e^2$, and comprises the correlated (R_H, R_L) data pairs from the $\nu = 1$ side of the transition. The dots that form the vertical line are from the $\nu = 2$ side of the transition, in the B range of 2.6–2.9 T in Fig. 1(b). In that B range R_H remains quantized at 0.5 h/e^2, while R_L can take any value in the range 0–0.5 h/e^2. The remaining, scattered, dots are mainly from the intermediate B range [2.9–3.1 T in Fig. 1(b)] of the transition between the QH states, and also include the (relatively few) deviations from the ordered lines. We note that the observed R_L-R_H relation is different from the derivative law relating the resistivity components observed in macroscopic samples [27–30].

Figure 2 also includes data obtained from the 20 and 100 μm Hall bars (green and blue dots, respectively). While the 20 μm dots exhibit similar behavior to those of the 2 μm sample, the 100 μm sample shows somewhat different characteristics. When $R_H > 0.7h/e^2$ the 100 μm dots are close to the $R_L + R_H = h/e^2$ diagonal line, but otherwise they form a continuous curve, with R_L values that are lower than the corresponding 2 and 20 μm R_L values, and do not split into either a diagonal or vertical line. We attribute this difference in the R_L-R_H dependence to the larger size of the 100 μm Hall bar. An infinite, homogenous, sample with the same aspect ratio as our samples is expected to have a semicircle R_L-R_H dependence as shown in dashed line in the figure [31,32]. Comparing the measured data with the semicircle trace we find that although the 100 μm Hall bar has the characteristics of a wider sample, it may not be large enough to exhibit the full semicircle behavior. This may be related to the fact that resistance fluctuations in the 100 μm Hall bar begin to be discernible at our lowest T.

The clear ordering of the (R_H, R_L) pairs evident in Fig. 2, and the fact that data from different size samples fall on top of each other, are surprising from several respects. First, the $\nu = 2$-1 transition does not take place at the same B range in all samples, due to small differences in electron density. Second, the apparent B width of the transitions, although not clearly defined due to the large fluctuations present in the 2 and 20 μm samples, varies between samples of different widths and is larger for the narrower samples (see Fig. 4 and discussion below) [33,34]. And third, the random nature of the fluctuations, unique to each sample and cooldown, indicates that a fundamental mechanism underlies the appearance of order in the data of Fig. 2.

The theoretical model that is most suitable for discussing transport in mesoscopic samples in the QH regime is the edge-state model [26,35,36]. In this model the electrons move along one-dimensional channels that follow the edges of the sample, with the direction of their motion set by the polarity of B. The resistance of the sample can be determined, following the Landauer formulation, by the probabilities of an edge-state electron to be transmitted forward along the same edge or reflected to a different edge of the sample. Using this approach, Streda *et al.* [24] and Büttiker [25] were able to derive explicit formulas for the resistances in the QH regime.

An intriguing result that directly stems from the Landauer analysis of QH samples was pointed out by Streda *et al.* [24]. They explicitly calculated R_L and R_H and showed that they obey the simple sum rule [37]

$$R_L + R_H = R_{2t}. \tag{2}$$

To test this prediction we plot, in Fig. 3, R_{2t} ($R_{63,63}$, black line) of the 2 μm Hall bar together with the sum $R_L + R_H$ (blue line) of the resistances from Fig. 1(b). R_{2t} is plotted after subtracting a B-independent contact resistance of 1 402 Ω, chosen by requiring that R_{2t} will be equal to R_H deep in the $\nu = 1$ QH state. As can be seen, the agreement between our data and Eq. (2) is very good, and includes the overall shape of the resistance trace between the $\nu = 2$ to the $\nu = 1$ QH states as well as most of the fluctuations.

The simple sum rule expressed by Eq. (2), together with its verification in Fig. 3, may seem, at first glance, a natural consequence of Kirchhoff's law. This becomes clear if we rewrite Eq. (2) as $(V_{65} + V_{53})/I_{14} = V_{63}/I_{14} = V_{63}/I_{63}$, and remember that we use the same value of current, $I_{14} = I_{63} = 1$ nA, for both measurements. However, we must keep in mind that the current paths, and the measurement geometry, in the two measurement configurations are different, and therefore the second equality in the equation above should not hold. Wider B-range measurements of our 2 μm sample, shown in the inset to Fig. 3, indicate that the sum rule of Eq. (2) is clearly violated near $B = 0$ and also where QH features are not fully developed. We can therefore regard Eq. (2) to be a special property of the QH regime.

In Fig. 3 we have shown that the simple sum rule of Eq. (2) predicted by Streda *et al.* [24] holds over the entire range of B covering the $\nu = 2$ and $\nu = 1$ QH states and the transition region between them. We have also shown that, over a limited, but large, B range, our data obey the experimentally derived Eq. (1). The coincidence of the two relations is where $R_{2t} = h/e^2$. An intriguing question is why the quantization of R_{2t} is maintained over a much broader range of B than the quantization of the four-terminal R_H (and the vanishing of R_L).

While we are unable to answer this question, we wish to remark on an additional difficulty that arises from this observation. This difficulty is related to the determination of the transition width, along with its T dependence, which are key parameters in the description of QH transitions [33,34,38,39]. In order to define the transition region for mesoscopic samples one fits a smooth function to the fluctuating data and obtains the width from this fit.

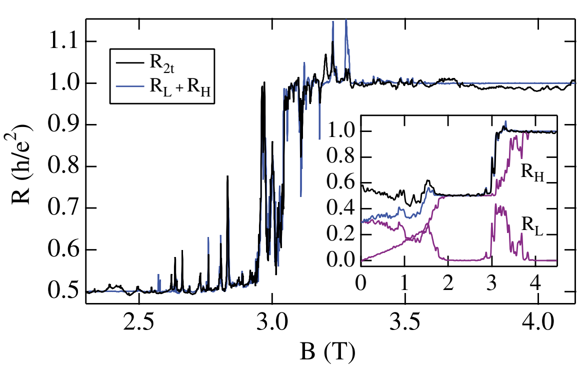

FIG. 3 (color). R_{2t} and $R_L + R_H$ of the 2 μm Hall bar in the vicinity of the $\nu = 2$-1 transition, $T = 10$ mK. The R_{2t} trace is shown after a subtraction of 1 402 Ω. Inset: R_L and R_H (purple), R_{2t} and $R_L + R_H$ over a wider B range. Note that $R_L + R_H \neq R_{2t}$ for $B < 2$ T.

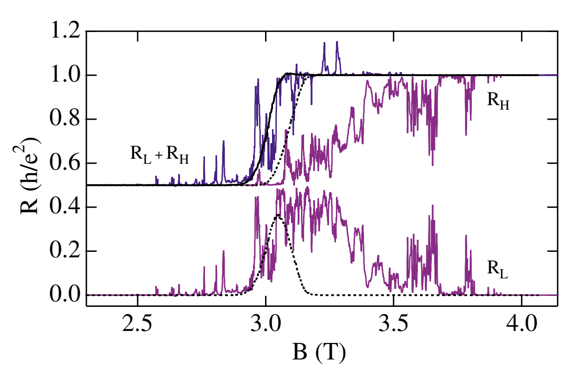

FIG. 4 (color). R_L and R_H (purple) and $R_L + R_H$ (blue) of the 2 μm Hall bar, together with R_L and R_H (dashed black) and $R_L + R_H$ (solid black) of the 100 μm Hall bar, in the vicinity of the $\nu = 2$-1 transition, $T = 10$ mK. The 100 μm traces are shifted to the left by 0.074 T.

If we apply this procedure to R_H and R_L that are obtained from the 2 μm sample, we find a much broader transition than the corresponding transition in the 100 μm sample. If, on the other hand, we use R_{2t} or, equivalently, the combination $R_L + R_H$ the resulting width is similar to that obtained from the 100 μm sample. This is illustrated in Fig. 4, where we plot R_L, R_H, and $R_L + R_H$ of the 2 and 100 μm Hall bars at the vicinity of the $\nu = 2$-1 transition. R_L and R_H of the 100 μm Hall bar display very small fluctuations and their transition region is narrower than that reflected from R_L and R_H of the 2 μm Hall bar. In contrast, for both samples, the sum $R_L + R_H$ has approximately the same width.

Generally, in mesoscopic samples, measurements that use different contact configurations yield different average resistance and fluctuation patterns. In our samples we find that the different resistance measurements are related in a way that is consistent with the R_L-R_H correlations discussed above. All of the measurements that were presented thus far were done using the contact configuration of $R_L = R_{14,65}$ and $R_H = R_{14,53}$. The relations to other contact configurations can be summarized as follows: each possible contact configuration of R_H ($R_{14,62}$ or $R_{14,53}$) results in a different resistance and fluctuation pattern. To each of these two options there corresponds one correlated R_L configuration ($R_{14,23}$ or $R_{14,65}$, respectively): $R_{14,65} + R_{14,53} = R_{14,23} + R_{14,62} = R_{63,63}$. When reversing the B polarity R_H changes sign and the corresponding R_L configuration is switched to the other side of the Hall bar: $R_H(-B) = -R_H(B)$ and $R_{14,23}(\mp B) = R_{14,65}(\pm B)$ [40].

To conclude, we have shown that in the QH regime $R_L + R_H = R_{2t}$, in agreement with the results of the model of edge-state conduction. For the B range where $R_{2t} = h/e^2$ this leads to R_L-R_H correlations of the form $R_L + R_H = h/e^2$. We have pointed out difficulties in estimating the width of QH transitions in mesoscopic samples.

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*Present address: Departamento de Física, Facultad de Ciencias, Universidad de Salamanca, E-37008 Salamanca, Spain

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