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QUANTUM HALL TRANSITIONS IN MESOSCOPIC SAMPLES

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We present an experimental study of four-terminal resistance fluctuations of mesoscopic samples in the quantum Hall regime. We show that in the vicinity of integer quantum Hall transitions there exist two kinds of correlations between the longitudinal and Hall resistances of the samples, one on either side of the transition region.

Keywords: Quantum Hall effect; mesoscopic samples; resistance fluctuations.

1. Introduction

The study of the relationship between the longitudinal and Hall transport coefficients in the quantum Hall (QH) regime has yielded fascinating results in the past twenty years. First came the phenomenological "derivative law" relating the longitudinal resistivity, ρ_{xx}, to the magnetic-field (B) derivative of the Hall resistivity, ρ_{xy}.[1–4] Somewhat later a different relation was theoretically proposed, and experimentally demonstrated.[5,6] predicting a "semicircle" relation between the longitudinal and Hall conductivities (σ_{xx} and σ_{xy}).[7] Incidently, these two relations are mathematically inconsistent with each other.

In this paper we report on the observation of new relations between the longitudinal and Hall resistance components (R_L and R_H) obtained from mesoscopic samples, in which finite-size effects are clearly observed. In addition to the familiar transport features of the QH states, and the transitions between them, our samples exhibit large reproducible resistance fluctuations. The R_L-R_H relationships we observed can be summarized as follows: For a transition between a $\nu = i$ to a $\nu = i-1$ QH state (here ν is the Landau level filling-factor), on the $\nu = i$ side of the transition $R_H = h/ie^2$ while R_L is non-zero and fluctuating, whereas on the $\nu = i-1$

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side of the transition $R_L + R_H = h/(i-1)e^2$. This last relation holds for the fine details of the mesoscopic fluctuations in our samples.

In order to limit our study to well-separated integer QH transitions and avoid complications arising from fractional QH states, we chose to use low-mobility samples. Our samples display a series of integer QH states down to the $\nu = 1$ QH state. As B is increased beyond $\nu \approx 1/2$ the samples turn insulating, with R_L that increases sharply with increasing B or decreasing temperature (T). This transition, from the $\nu = 1$ QH state to the insulating phase ($\nu = 1-0$ transition), is considered to be the fundamental integer QH transition since it only involves the lowest Landau-level. We shall therefore separately consider, and compare, our results from studying the transport in the lowest Landau-level and those obtained from higher Landau-level transition. As expected, we find no fundamental differences.

2. Samples and measurements

Our samples were prepared from two InGaAs/InAlAs wafers that contain a 200 Å quantum-well. The samples were wet-etched to a Hall-bar geometry with a lithographic width of $W = 2$ μm and a voltage-probe separation of 4 μm. Two samples, T2C and T1B, were cooled in a dilution refrigerator with a base T of 10 mK. Sample T2C was cooled twice (T2Ci2 and T2Cm2), and had a density and mobility of $n_s = 1 \times 10^{11}$ cm^{-2}, $\mu = 13,700$ cm^2/Vs and $n_s = 1.15 \times 10^{11}$ cm^{-2}, $\mu = 14,000$ cm^2/Vs for T2Ci2 and T2Cm2, respectively. Sample T1B was cooled once (T1Bc2), and had $n_s = 3.65 \times 10^{11}$ cm^{-2}, $\mu = 44,000$ cm^2/Vs.

Four-terminal resistance measurements were done using standard ac lock-in techniques with frequencies of $1-4$ Hz and a current $I = 0.1 - 1$ nA. Due to the small size of our samples, their resistances display reproducible fluctuations whose magnitude and B-correlations near $B = 0$ were used to extract the phase-coherence length of the electrons, L_ϕ[8]. For our samples $L_\phi = 1.1 - 1.3$ μm at $T = 10$ mK.

3. Transport in the lowest Landau level

In Fig. 1 we show the resistance of sample T2Ci2 in the vicinity of the $\nu = 1-0$ transition. We plot R_L and the antisymmetric part of R_H, R_H^a[9], against B. The dashed line is at the approximate B-value where the transition occurs, as determined from the T-dependence of R_L (see inset). On the low-B side of the transition the sample is in the $\nu = 1$ QH state, with $R_H^a = h/e^2$ and R_L vanishingly small. As B is increased beyond $\nu = 1$, the two transport components are seen to behave in a sharply different manner. R_L increases, reaching 25 h/e^2 on the insulating side, and displays large fluctuations that are of the same order as its mean value. At the same B range, R_H^a appears to be free of fluctuations, remaining close to its quantized value of h/e^2 for the entire B-range of the transition.

This state of the electronic system, where R_L is non-zero and displays an insulating behavior while at the same time R_H^a remains quantized, was previously observed in large, macroscopic, samples[6,10,11] and was termed a "quantized Hall

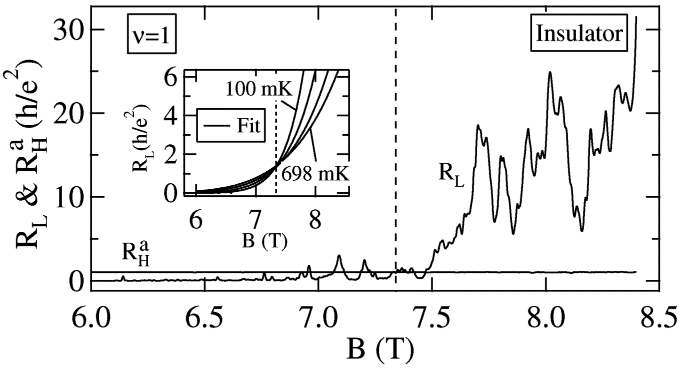

Fig. 1. R_L and R_H^a of sample T2Ci2 in the vicinity of the $\nu = 1 - 0$ transition, $T = 100$ mK. Inset: Smooth fits to R_L traces, used to distinguish the QH and insulating regimes.

insulator" (QHI). Models of transport at high B[12], based on conduction through a random Chalker-Coddington-like network of puddles, have produced the same kind of behavior but did consider directly the possibility of mesoscopic fluctuations. Our findings show that a QHI is also found in mesoscopic samples, even when R_L is dominated by strong fluctuations[13].

4. Transport in higher Landau levels

In Fig. 2(a) we show R_L and R_H of sample T2Cm2 in the vicinity of the transition between the $\nu = 2$ and the $\nu = 1$ QH states. These two states are clearly resolved, having a quantized R_H and a vanishing R_L. The mesoscopic nature of this sample is evident in the transition region between the two QH states, where large reproducible fluctuations are seen both in R_L and R_H. A closer look at the fluctuations reveals unusual correlations between the transport components. These correlations are of two kinds, one on either side of the transition region. The two dashed lines in the figure mark the boundaries of the two corresponding B-ranges, as we explain below.

The first B-range is on the $\nu = 2$ side of the transition, to the left of the left dashed line. There the behavior of the two resistance components is similar to that found in the vicinity of the $\nu = 1 - 0$ transition: R_H is quantized to its $\nu = 2$ value, 0.5 h/e^2, displaying no fluctuations, while R_L is fluctuating between $0 - 0.5$ h/e^2. When B is increased beyond this range R_H ceases to be quantized and both R_L and R_H display fluctuations.

On the high-B side of the transition, to the right of the right dashed line in Fig. 2(a), there is different kind of relation between R_L and R_H. Both resistance components exhibit fluctuations, with a near-perfect correlation between the fluctuations of R_L and those of R_H. Graphically, we observe that for each peak in R_L there corresponds a dip in R_H of nearly equal magnitude, and vice versa. Mathematically, these correlations can be expressed by $R_L + R_H = h/e^2$.

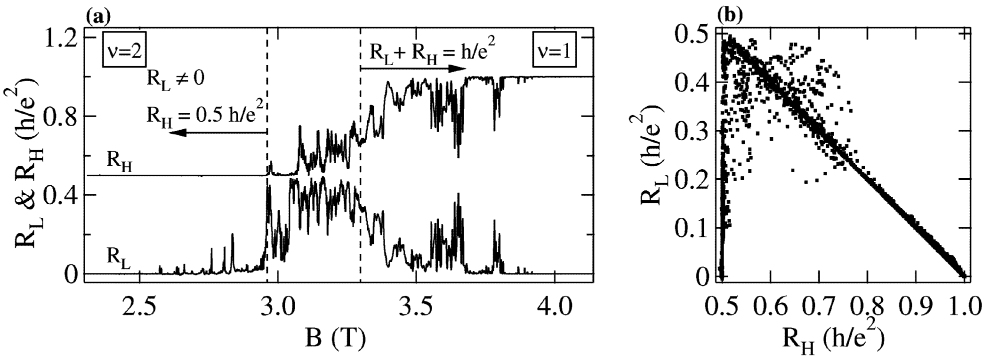

Fig. 2. The resistance of sample T2Cm2 in the vicinity of the $\nu = 2 - 1$ transition, $T = 10$ mK. (a) R_L and R_H *vs.* B. (b) R_L *vs.* R_H.

The two kinds of R_L-R_H dependences can be seen more clearly in Fig. 2(b), where we present a plot of R_L *vs.* R_H. In this unconventional plot[14] each dot represents one (R_H, R_L) data-pair from the B traces of Fig. 2(a). One can see that, aside from some scatter, the dots fall into two ordered groups: a diagonal line stretching from $(0.5\ h/e^2, 0.5\ h/e^2)$ to $(h/e^2, 0)$ and a vertical line at $R_H = 0.5\ h/e^2$. The diagonal line corresponds to $R_L + R_H = h/e^2$, and is comprised of the correlated (R_H, R_L) data-pairs from the $\nu = 1$ side of the transition. The dots that form the vertical line are from the $\nu = 2$ side of the transition. The remaining, scattered, dots are mainly from the intermediate B-range that is between the two dashed lines in Fig. 2(a), and also include the (relatively few) deviations from the correlated behavior. This clear ordering of the data is found in all cooldowns of sample T2C despite the random nature of the fluctuations which are unique to each cooldown.

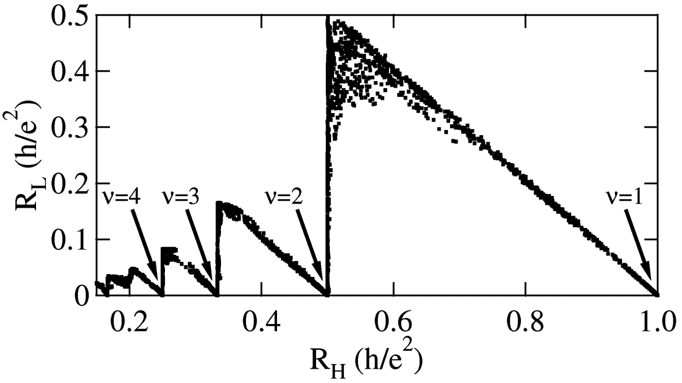

Fig. 3. R_L *vs.* R_H for sample T1Bc2, $T = 10$ mK. The positions of the QH states are marked by their corresponding filling factor numbers.

To check whether this behavior is common to other QH transitions we repeated our measurements with a higher density sample, T1Bc2, allowing us to observe well-separated $\nu = 4-3$, $3-2$, and $2-1$ QH transitions. In Fig. 3 we plot R_L *vs.* R_H for this sample. The saw-tooth shape that is formed by the $(R_H,\, R_L)$ pairs indicates that the same kind of R_L-R_H dependence is present in all transitions. The vertical lines in the figure, present at $R_H = h/ie^2$ with i =4, 3, and 2, are formed by data points from the low-B side of the transitions, where R_H is quantized while R_L is fluctuating. The diagonal lines correspond to data points from the other, high-B, side of the transition, where R_L and R_H exhibit fluctuations of equal magnitude and opposite sign, such that their sum equals the resistance value of the next QH plateau, $R_L + R_H = h/(i-1)e^2$, again with i =4, 3, and 2.

5. Discussion

We now turn to discuss the possible origins of the observed R_L-R_H dependence. The accepted theoretical model for describing transport in the QH regime is the edge-states model[15]. In the transport models of Streda, Kucera and MacDonald[16] and of Jain and Kivelson[17] resistance fluctuations appear as a result of electrons, moving along the edges of the sample, being scattered between one side of the sample to the other. When only one edge state is present, corresponding to conduction via the lowest Landau level alone, these models predict that the fluctuations will be limited to R_L, leaving R_H quantized. This situation is in agreement with our observations at the $\nu = 1-0$ transition, but does not account for the quantized R_H on the low-B side of the higher Landau level transitions of $\nu = 4-3$, $3-2$, and $2-1$.

An interesting prediction in Ref. 16 is that the sum of R_L and R_H should be equivalent to a two-terminal resistance of the sample (R_{2t}), $R_L + R_H = R_{2t}$. In a previous work[18] we have verified this equality in the QH regime. The anti-correlation between R_L and R_H, that is present on the high-B side of the QH transitions, occurs in a B-range in which R_{2t} is quantized. An intriguing question is why the quantization of R_{2t} is maintained over a much broader range of B than the quantization of the four terminal R_H or the vanishing of R_L.

In a recent numerical simulation[19] Zhou and Berciu make use of the edge-state formulation to describe the resistance in the QH regime as a result of an interplay between chiral edge-currents and the tunneling between opposite edges of a Hall-bar. Their simulations reproduce many of the central features of our results, identifying a low- and high-B regions for all QH transition.

Finally, we wish to note that in general the resistance of a mesoscopic sample, and its pattern of fluctuations, is unique to the contact configuration and B polarity used in the measurement. A full understanding of the properties of R_L and R_H therefore requires a comparison of different measurement setups. This issue is addressed in a separate publication[20].

6. Conclusions

We presented an experimental study of the relation between the resistance components of mesoscopic samples in the QH regime. We showed that there exist two kinds of relations between R_L and R_H in the vicinity of integer QH transitions, one on either side of the transition region. For a transition between a $\nu = i$ to a $\nu = i - 1$ QH state our findings can be summarized as follows: On the low-B (high-ν) side of the transition there is a B-range in which R_H is quantized to its value on the preceding QH plateau, $R_H = h/ie^2$, while R_L is non-zero and fluctuating. This also includes the low-B side of the transition to the insulating phase. On the high-B (low-ν) side of the transition R_L and R_H are anti-correlated, exhibiting fluctuations of equal magnitude and opposite sign, such that their sum equals the resistance value of the next QH plateau, $R_L + R_H = h/(i-1)e^2$.

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